On Formants

R. S. McGowan

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for my teachers,

Louis Goldstein, Michael Howe, and Philip Rubin

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Preface

Formants are "...one of several characteristic bands of resonance, a combination of which determines the distinctive sound quality of a vowel..." according to the *Oxford English Dictionary*. The first use of the term formant was in an 1894 German publication, and it first appeared in English in a 1901 issue of *Nature*. The term has been used to refer to any spectral prominence, but we restrict ourselves to resonances of the vocal tract. Also we recognize that speech segments other than vowels are, at least partly, characterized by their formants. Because formants and resonances are taken to be synonymous here, each formant is associated with a frequency, its formant frequency, which is simply a resonance frequency.

In fact, the magnitudes of raising and lowering of formant frequencies from their values, say, for a neutral vocal tract shape, are very important ways that sounds of speech are characterized. In this book, we consider the causal relation between cross-sectional areas of a tube and the first two formant frequencies. Tubes are employed to explain the acoustic properties of a vocal tract. Cross-sectional areas are the areas of the tube in planes perpendicular to the tube axis. If cross-sectional areas are associated with positions along the tube axis, we obtain an area function. Understanding the relationship between tube area functions and formants is an intermediate step toward understanding the causal relationship between articulation and acoustics.

Among the findings presented in this book are 1) the axial length of a constriction has a large effect on formant frequencies, 2) that a mathematical object, spatial phase, is an important concept in understanding how formant frequencies change with changes in tube cross-sectional area, and 3) it appears possible to characterize monophthong vowel production in an abstract form of area function parameters. Chapters and sections that may be omitted on a first reading of this book are marked with an * in the Table of Contents.

The findings in the present book rely on the results published in *Acoustics of Speech Production* (McGowan 2018). A short introductory chapter may enable the reader to understand the present book, however

a more compete understanding of the results presented here can be obtained by reading McGowan (2018).

I have many people to thank for their consistent encouragement regarding this work. These people include Richard Goldhor, Joel MacAuslan, Terry McKiernan, Susan Nittrouer, and Mark Tiede. This book is dedicated to my three most important teachers in phonetics, fluid mechanics and acoustics, and writing: Louis Goldstein, Michael Howe, and Philip Rubin.

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